

# **Evaluating the Efficiency of Regulation in Matching Markets with Distributional Disparities**

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# introduction

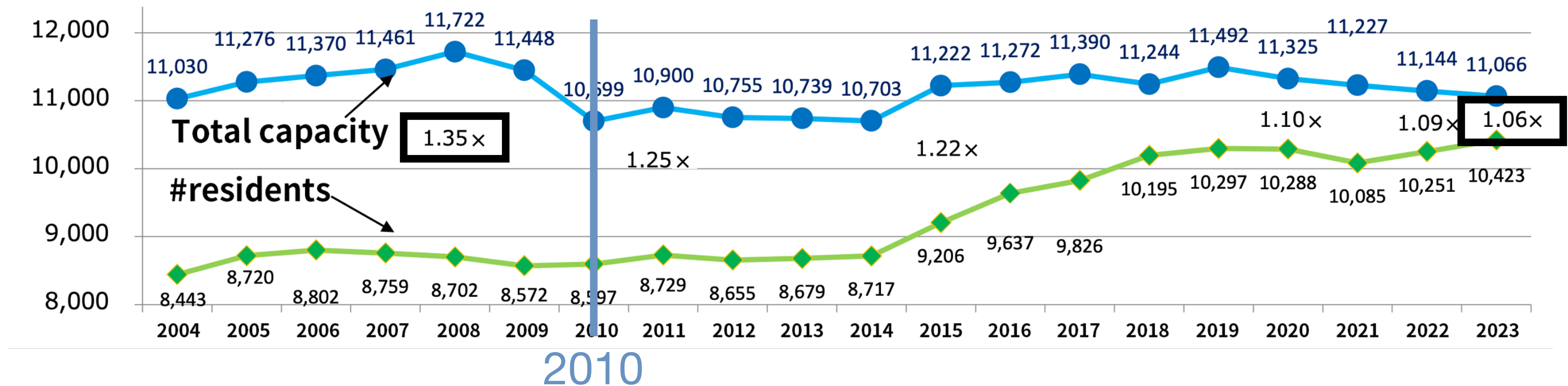
- outcomes in matching markets often diverge from socially desirable outcomes
- policymakers often intervene in the market
  - e.g., affirmative action in school choice, gender quotas in elections
- a common intervention is **cap-based regulation**
  - restricts the number of matches for certain categories

# Japan Residency Matching Program (JRMP)

- students and hospitals are matched for a two-year residency program
  - **10,000** students from 80 schools
  - 11,000 positions from 1,000 hospitals
- without intervention, **rural areas remain underserved**
  - in 2009, **48.6%** of residents were matched with one of the **6** (out of **47**) prefectures
- policymakers want to ensure adequate coverage of residents in all areas

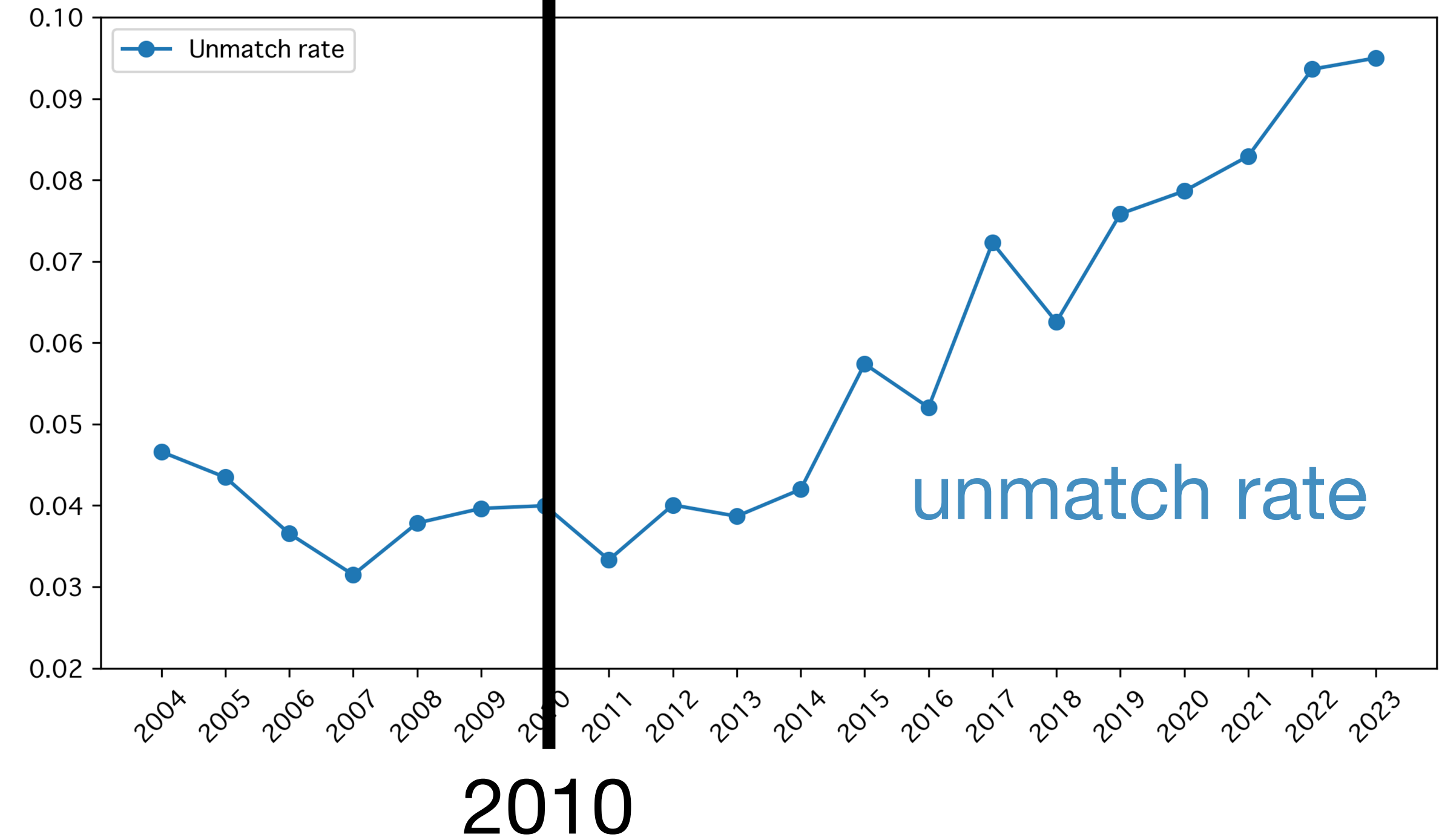
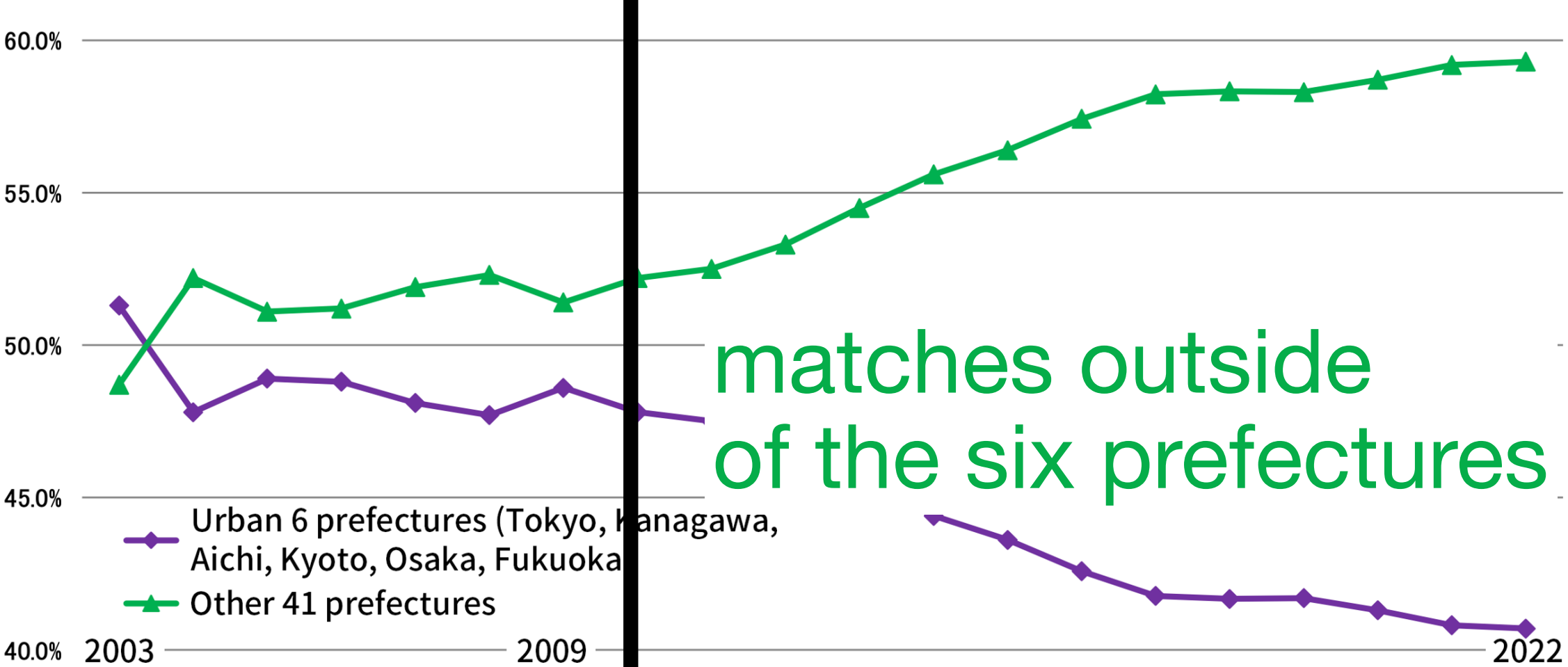
# cap-based regulation in JRMP

- government has enforced a cap-based regulation **since 2010**.



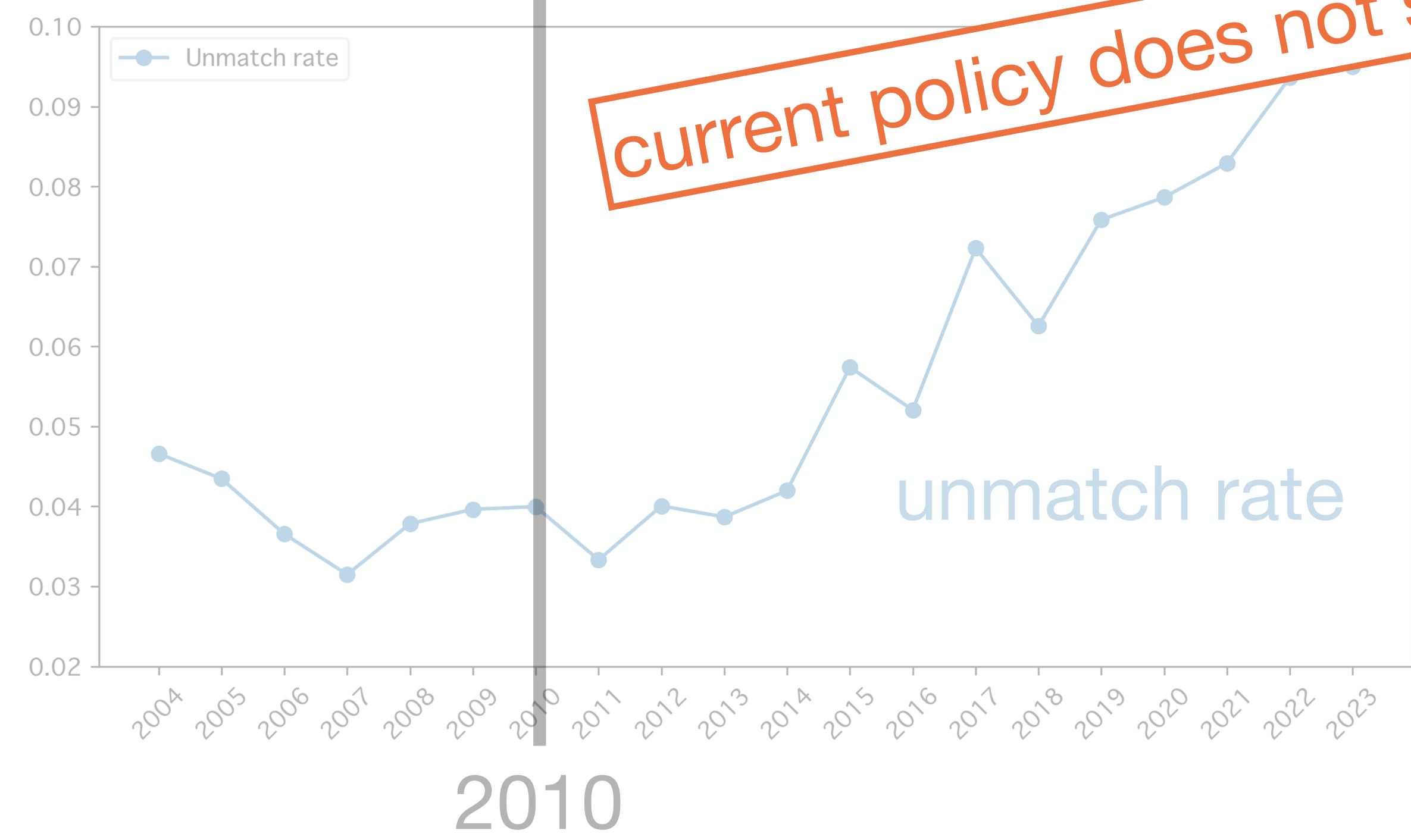
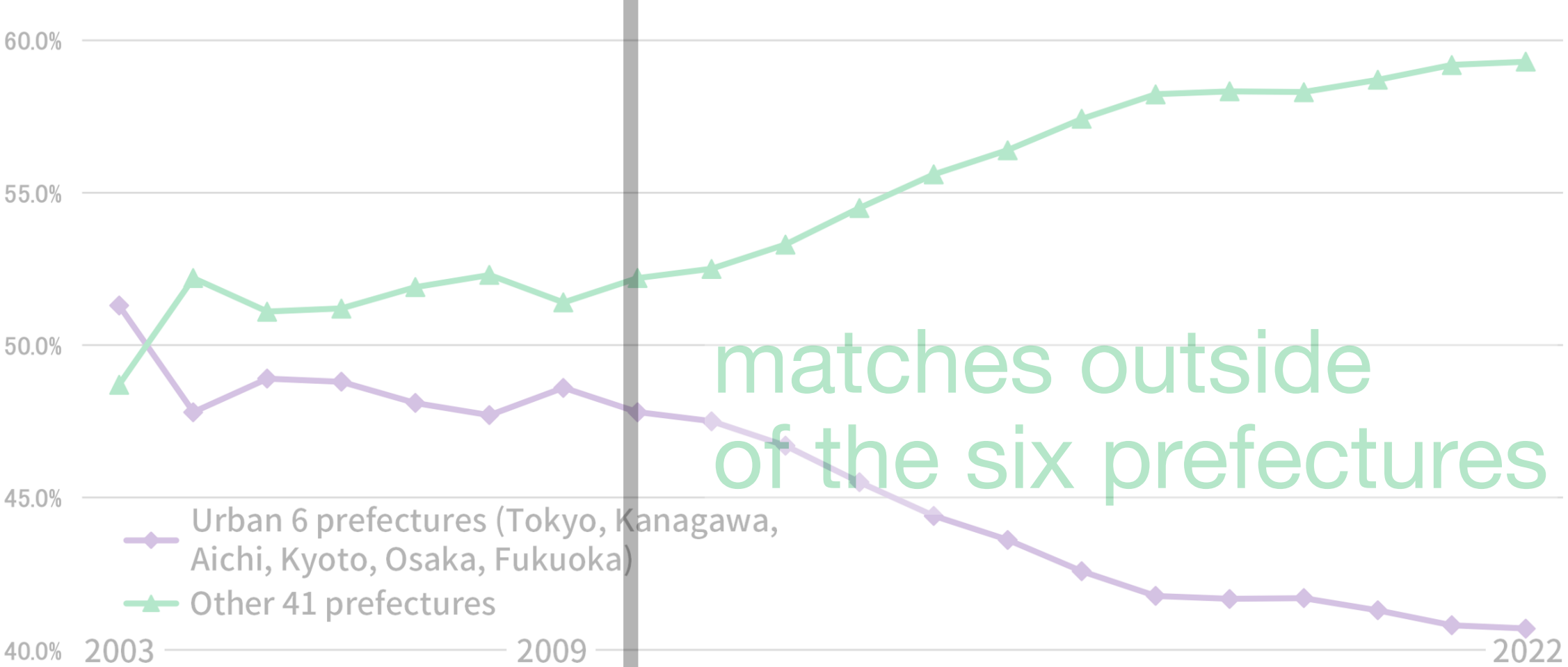
- ratio of positions to residents** is decreasing over time. (**1.35** in 2008 to **1.06** in 2023)
- positions in urban areas** have been reduced significantly

# results of the regulation



- matches in rural areas have increased (+)
- significant geographic disparity still remains (-)
- unmatch rate has been increasing (-)

# results of the regulation



current policy does not seem very successful

- matches in rural areas have increased (+)
- significant geographic disparity still remains (-)
- unmatch rate has been increasing (-)

# questions

- how effective is cap-based regulation?
- how do alternative policies, such as monetary interventions, compare?
- can we quantify their performance?



# contribution

this paper:

- develops a framework to evaluate policies in matching mkt with distributional constraints
  - transferable utility matching model with **regional constraints**
  - optimal taxation policy outperforms any cap-based policy
  - optimal taxation policy can be computed using data
- applies the framework to a novel dataset from the Japan Residency Matching Program:
  - status quo cap-based regulation generates a **significant welfare loss**
  - modest subsidy can achieve the same distributional goal, improving welfare



# some relevant literature

- **matching with distributional constraints**
  - **Kamada and Kojima (2015)**, Abdulkadiroglu and Sonmez (2003), Ehlers, Hafalir, Yenmez, and Yildirim (2014), Kojima (2012), Hafalir, Yenmez, and Yildirim (2013), Fragiadakis and Troyan (2017) ...
  - uses the **non-transferable utility (NTU)** matching model
  - primarily focuses on how to **set caps**, adjusting the deferred acceptance algorithm
- **our paper:** uses the **transferable utility (TU)** matching model
  - accommodates a **broader class of policies**, including monetary interventions
  - accounts for **endogenous transfer** (e.g., salary adjustment in response to intervention)



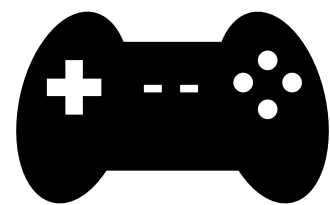
model



theoretical results



estimation

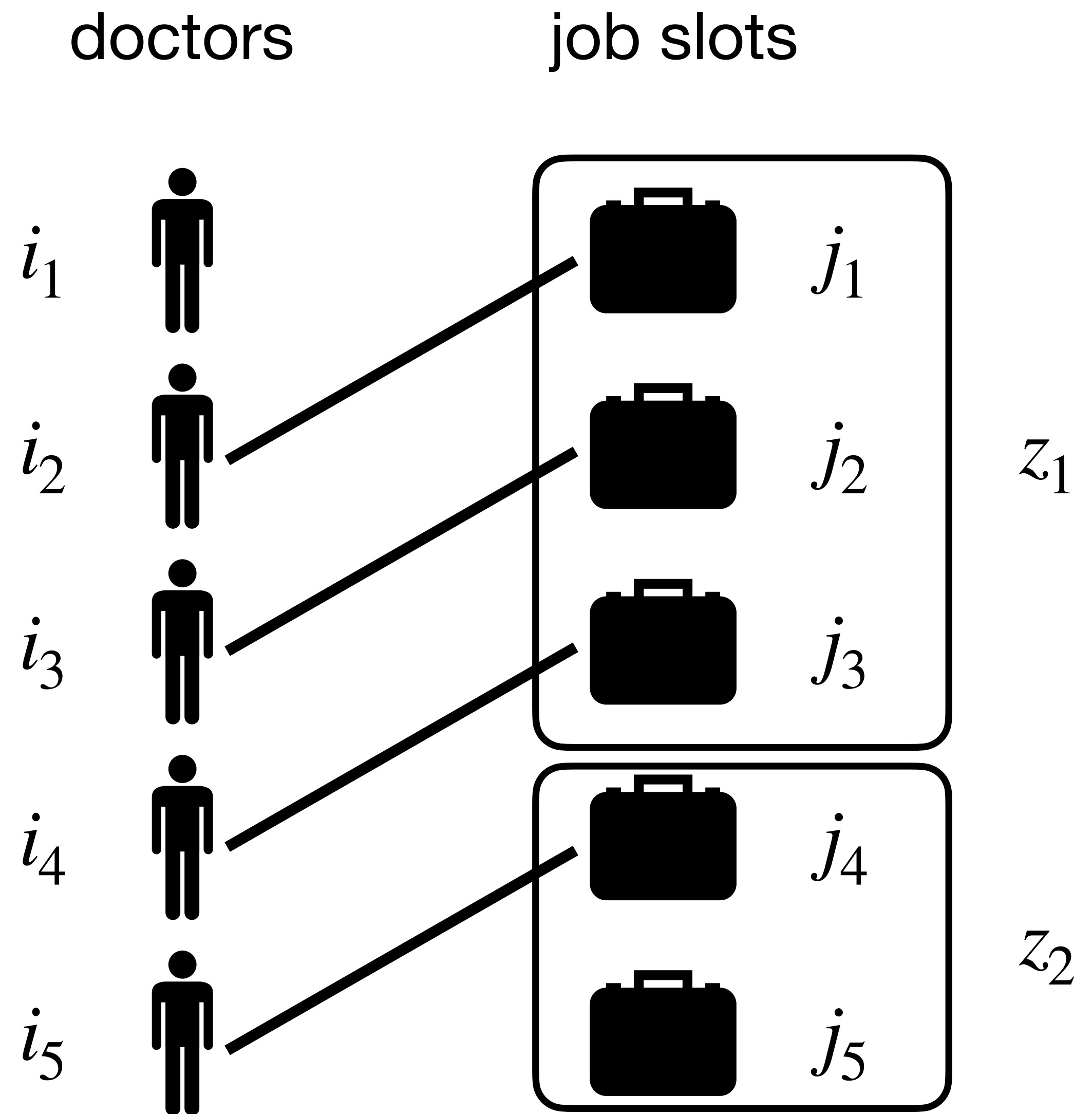


simulation

# setup

- **doctors**  $i \in I$  and **job slots**  $j \in J$
- each slot belongs to a **region**  $z \in Z$
- if  $i$  and  $j$  are matched, the pair generates a **joint surplus**  $\Phi_{ij} \in \mathbb{R}$  and splits it
- agents know  $(\Phi_{ij})_{i,j}$  and form the **matching**, achieving a "**stable outcome**"

no blocking pair + IR



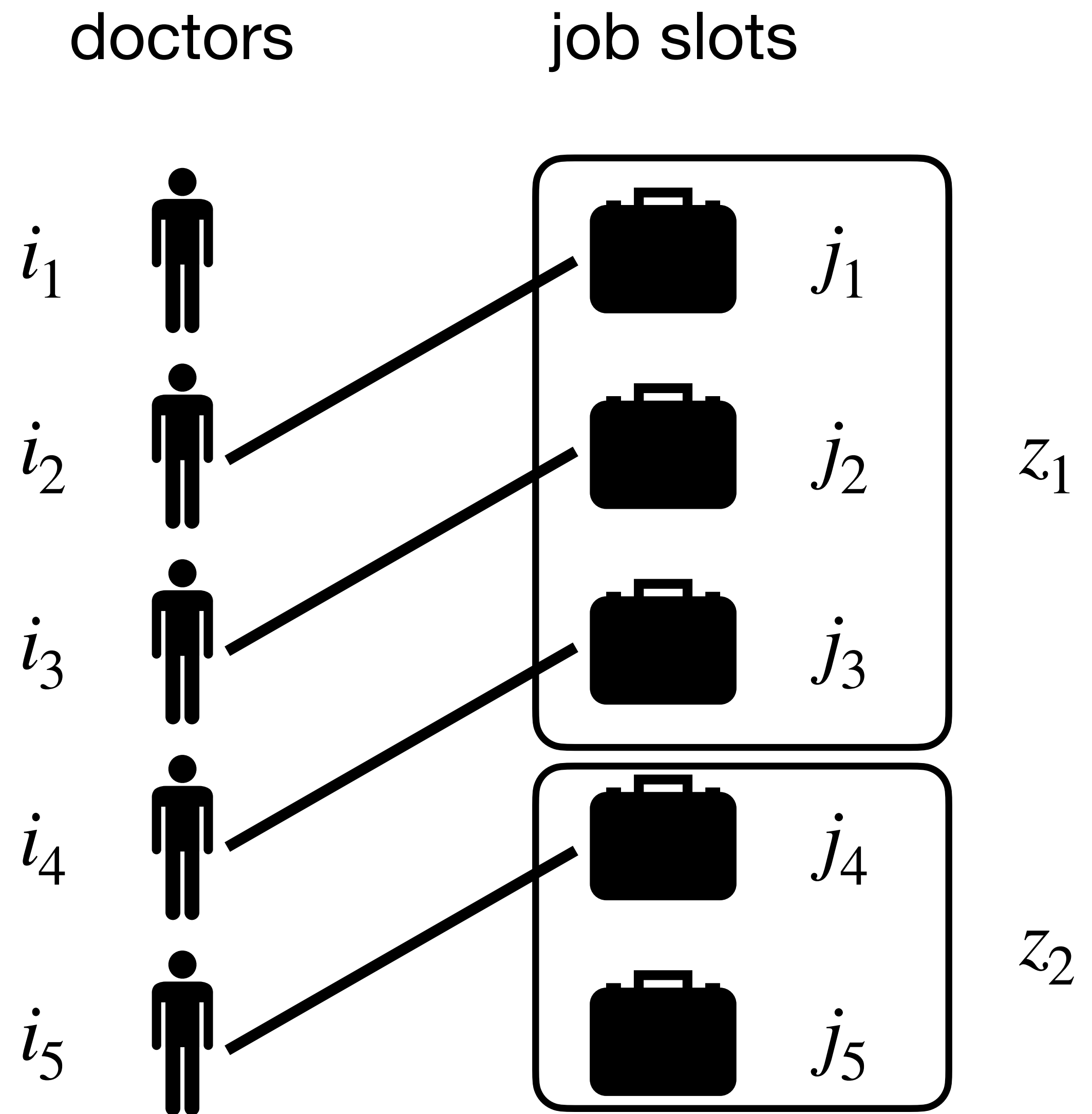
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doctor  $i$  and slot  $j$  **block** the matching if

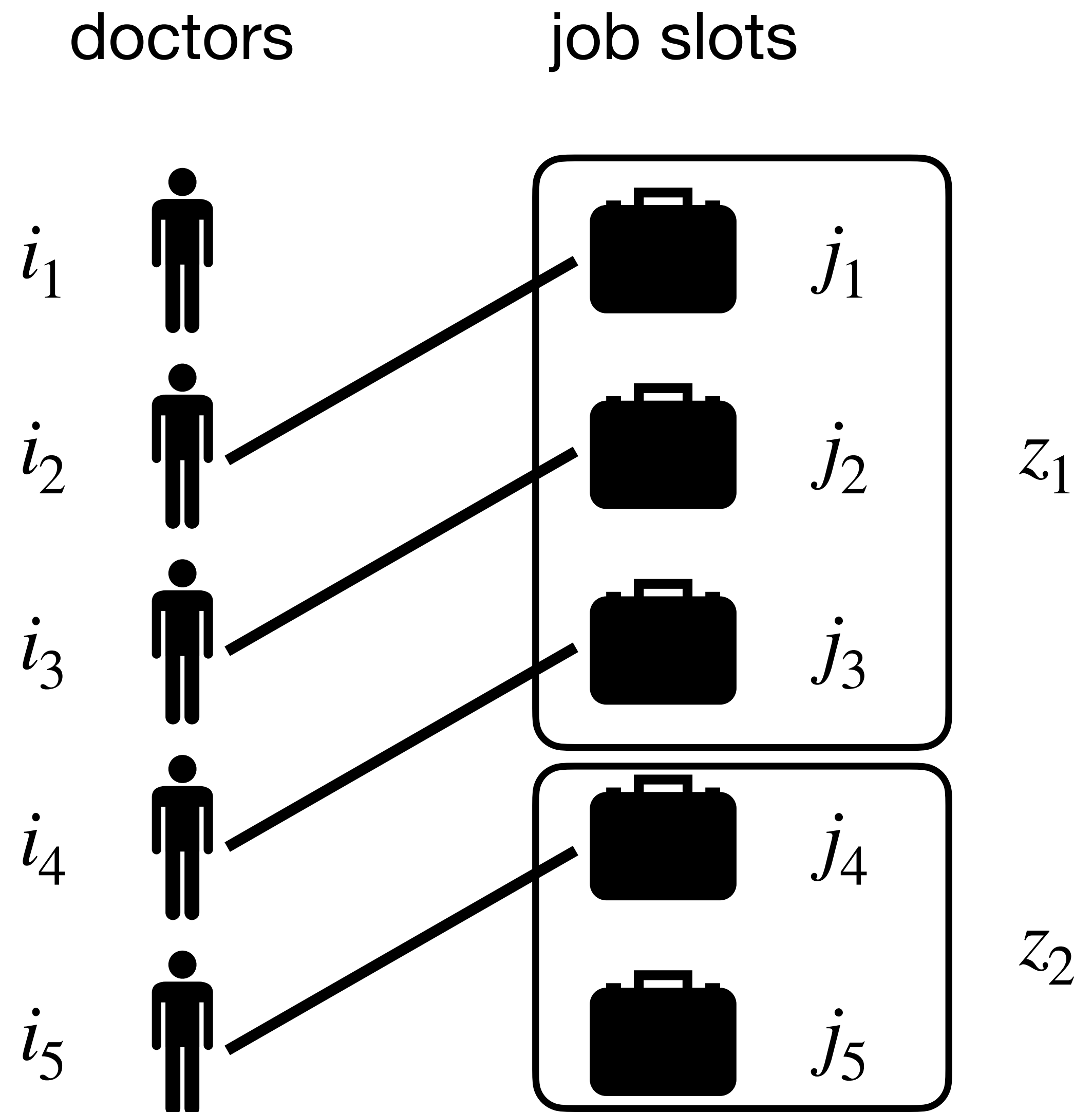
$$u_i + v_j < \Phi_{ij}$$

$u_i$   $i$ 's current payoff     $v_j$   $j$ 's current payoff



# setup

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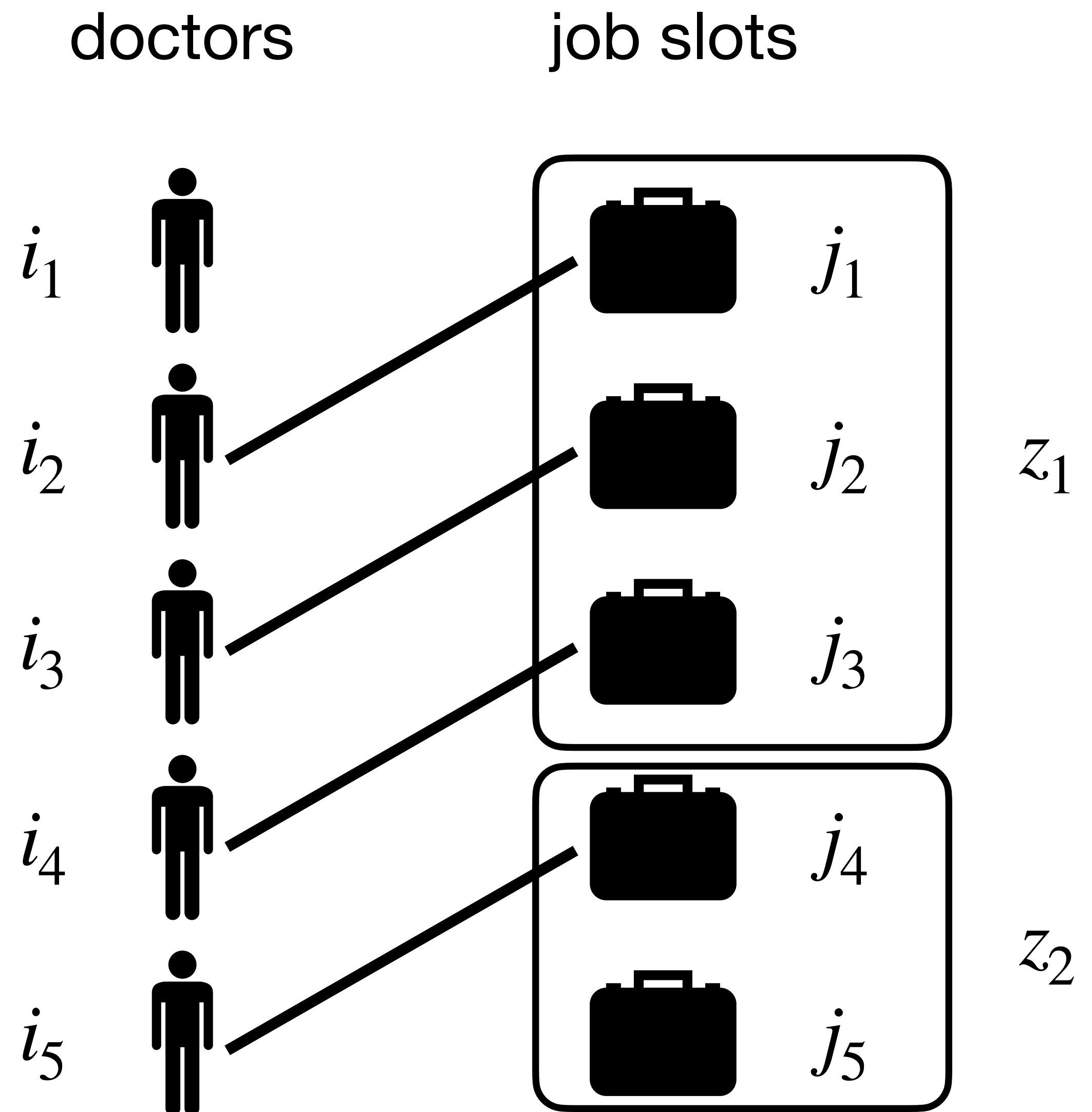
doctor  $i$  and slot  $j$  **block** the matching if

$$u_i + v_j < \Phi_{ij}$$

they can be better off by  
forming a new match and splitting the surplus

# setup

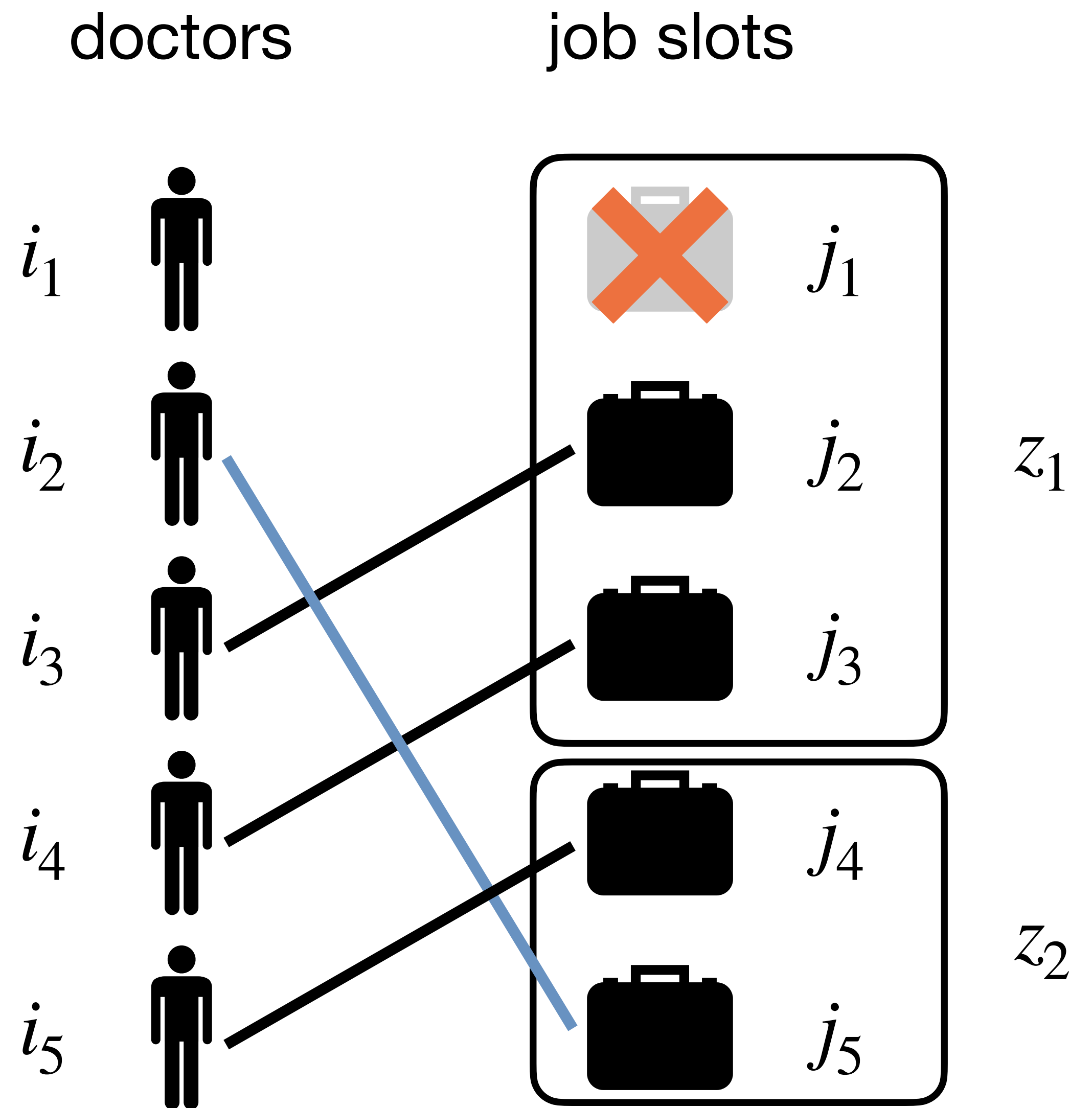
- policymaker (PM) faces exogenous **regional constraints**
  - **lower and upper bounds** on # of matches in each region
  - without intervention, the matching formed by the agents may not satisfy regional constraints
  - PM's goal: ensure # of matches in every region stays within the bounds



# of matches in  $z_2 \geq 2$

# cap-based policy

- **cap-based policy** removes positions in high-demand regions to encourage inflow into low-demand regions
- given the caps, agents form a stable outcome over the available slots



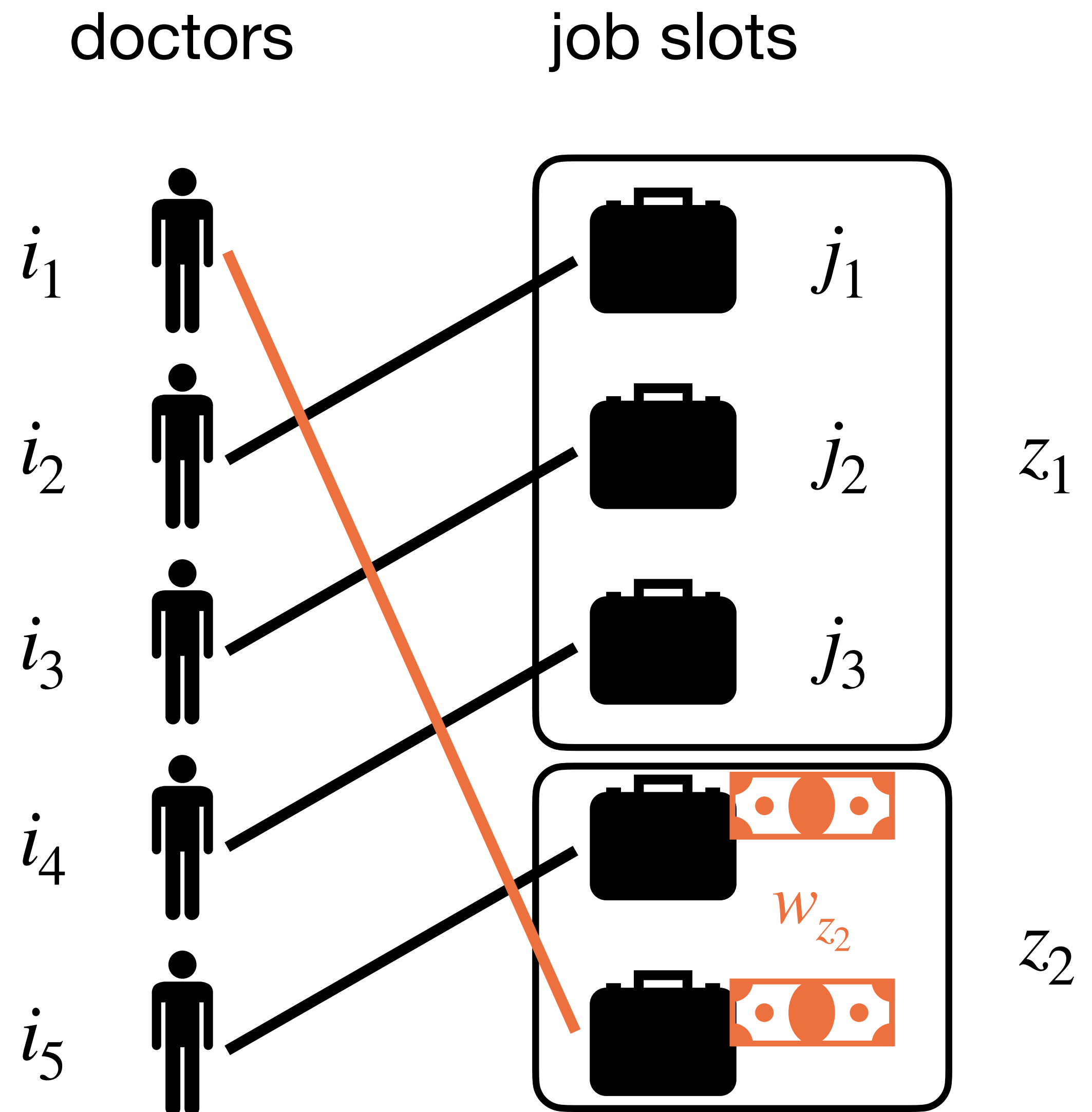
# of matches in  $z_2 \geq 2$



# taxation policy

- alternative: **taxation policy**
  - **tax**  $w_z \in \mathbb{R}$  is applied uniformly to all the matches in region  $z$ 

negative tax = subsidy
  - Given  $(w_z)_z$ , agents form a stable outcome as if **net joint surplus**  $\Phi_{ij} - w_z$  were the joint surplus
  - by choosing taxes properly, PM may induce socially desirable matchings



# of matches in  $z_2 \geq 2$



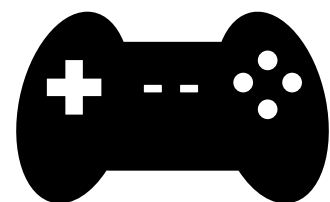
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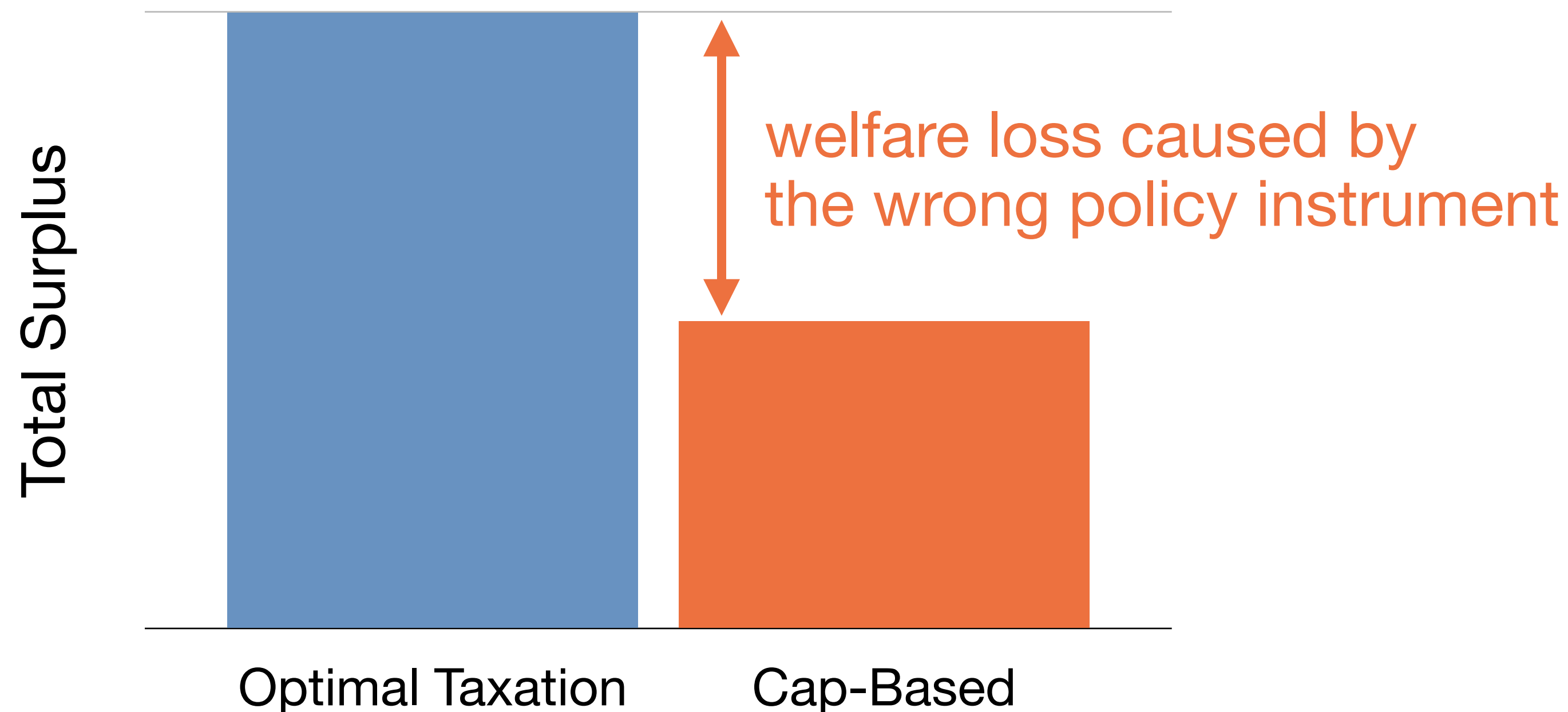


simulation

# optimal taxation policy

**Theorem 1 (informal):** optimal taxation policy generates a higher social surplus than any other cap-based policy that satisfies the same regional constraints.

- "taxation policy is better"
- the outcome under the optimal taxation policy serves as a benchmark:



# optimal taxation policy

(P<sub>0</sub>)

$$\begin{aligned}
 & \text{maximize}_{d \in \{0,1\}^{I \times J}} && \sum_{(i,j) \in I} \Phi_{ij} d_{ij} + \sum_{i \in I} \left(1 - \sum_{j \in J} d_{ij}\right) \Phi_{i,j_0} + \sum_{j \in I} \left(1 - \sum_{i \in I} d_{ij}\right) \Phi_{i_0,j} && \text{total surplus generated under } d \\
 & \text{subject to} && \sum_{j \in J} d_{ij} \leq 1 && \forall i \in I, \\
 & && \sum_{i \in I} d_{ij} \leq 1 && \forall j \in J, \\
 & && \underline{o}_z \leq \sum_{j \in z} \sum_{i \in I} d_{ij} \leq \bar{o}_z && \text{regional constraints} \quad \forall z \in Z,
 \end{aligned}$$

- optimal taxation policy  $w^*$ : **Lagrange multipliers (shadow prices)** for the regional constraints
  - can be computed by LP (we can relax the integrality constraint)
- $w^*$  induces a stable outcome with matching  $d^*$  that solves (P<sub>0</sub>)

# optimal taxation policy

(P<sub>0</sub>)

$$\begin{aligned}
 & \underset{d \in \{0,1\}^{I \times J}}{\text{maximize}} && \sum_{(i,j) \in I} \Phi_{ij} d_{ij} + \sum_{i \in I} \left(1 - \sum_{j \in J} d_{ij}\right) \Phi_{i,j_0} + \sum_{j \in J} \left(1 - \sum_{i \in I} d_{ij}\right) \Phi_{i_0,j} && \text{total surplus generated under } d \\
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 & && \underline{o}_z \leq \sum_{j \in z} \sum_{i \in I} d_{ij} \leq \bar{o}_z && \text{regional constraints} \quad \forall z \in Z,
 \end{aligned}$$

**Theorem 1 (formal):** if PM knows  $(\Phi_{ij})_{i,j}$ , then for any regional constraints, PM can compute a taxation policy  $(w_z^*)_z$  that induces the matching  $d^*$  that maximizes the total surplus subject to the regional constraints

# unobs. heterogeneity and aggregate-level data

- in practice, the joint surplus  $\Phi_{ij}$  is unknown to PM and often hard to identify
- suppose that PM has access to past **aggregate-level matching data**:
  - doctors' observable characteristics  $s \in S$  (**school**)
  - job slots' observable characteristics  $h \in H$  (**hospital**)
  - **# of matches**  $\mu_{sh}$  between school  $s$  and hospital  $h$

$$\mu = (\mu_{s,h})_{s,h} = \begin{matrix} & h_1 & h_2 & h_3 \\ \begin{matrix} s_1 \\ s_2 \end{matrix} & \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \end{matrix}$$

# taxation policy with aggregate-level data

- key assumption: **additive separability** (Galichon and Salanié, 2021):

$$\Phi_{ij} = \Phi_{sh} + \varepsilon_{ih} + \eta_{sj} \quad \text{indep. error terms}$$

aggregate-level  
joint surplus

**Theorem 2 (informal):** Assume additive separability and error distributions. Under certain regularity conditions, we can compute the optimal taxation policy using aggregate-level matching data  $\mu$ .





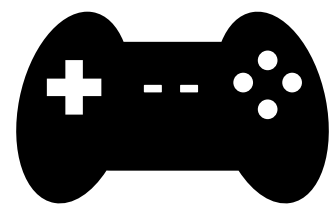
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estimation



simulation

# available data

from 2016 to 2019

- **JRMP matching outcomes**

- # of matches between all schools and hospitals
- # of positions offered in each hospital

- **Salary**

- monthly salary paid to residents by each program

- **Characteristics of hospitals and schools**

- **hospital:** # of beds, # of emergency transport cases
- **school:** private or public, T-scores (difficulty of entrance exam)
- **locations**

hospital size

students' quality

# two-step procedure

- we want to estimate agents' preferences to perform counterfactual simulations

$$u_i = U_{ij}^{\text{base}} + \tau_i$$

equilibrium payoff  
(actual payoff)

base utility

transfer


utility that an agent derives  
from the match without transfer

# two-step procedure

- we want to estimate agents' **preferences** to perform counterfactual simulations

$$u_i = U_{ij}^{\text{base}} + \tau_i$$

equilibrium payoff (actual payoff)    **base utility**    transfer

- we estimate these objects in two steps:
  1. estimate equilibrium payoff  apply GS2021's method
  2. estimate the parameters in the **base utilities and transfers**
- (see the paper for details of estimation methods and results)

# Step 2: more details

$$u_i = U_{ij}^{\text{base}} + \tau_i$$

- "transfer" in our application consists of many different components:
  - salary, workload, experience, risk of medical accidents, etc.
- we assume that agents' utilities are **quasi-linear** in monetary transfers
- we regress equilibrium payoffs on agents' characteristics and salary (+IV) to estimate agents' base utility and marginal value of salary
  - we can evaluate the agents' utilities and social welfare in terms of money

unobservable



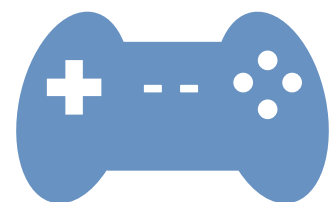
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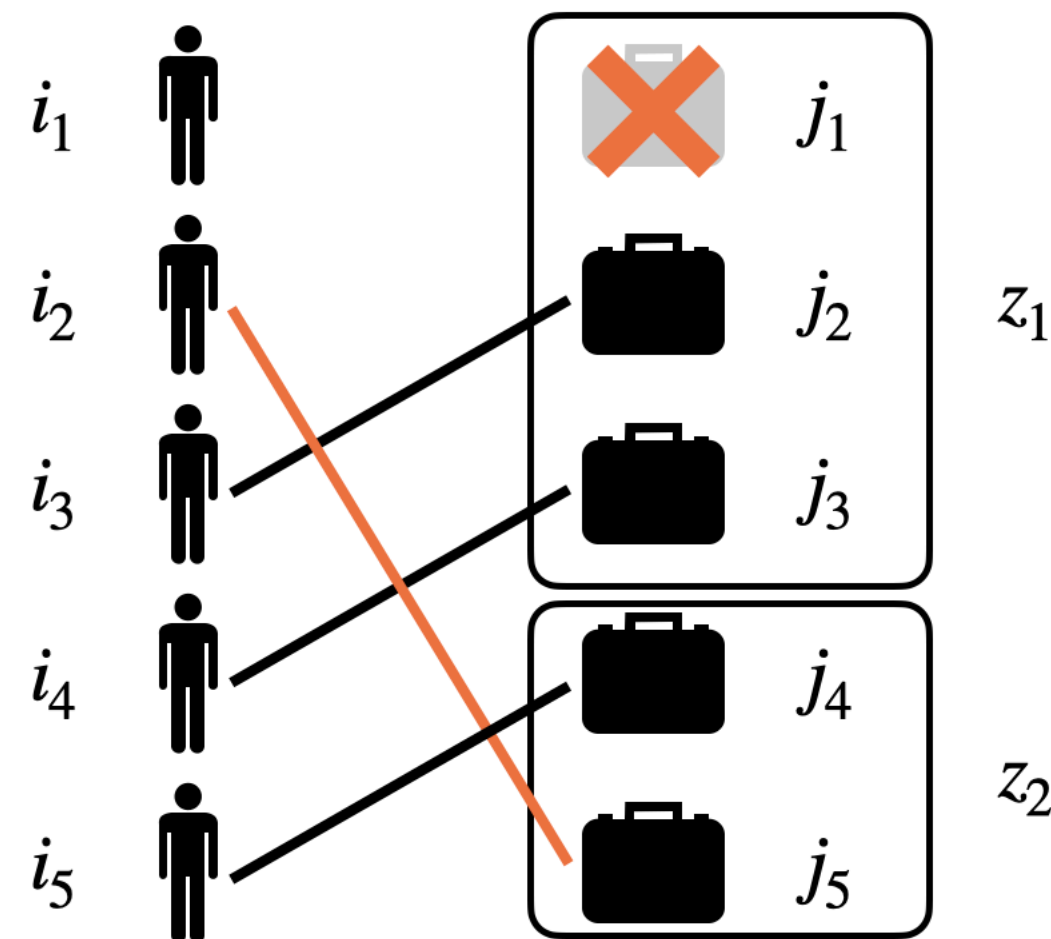


simulation

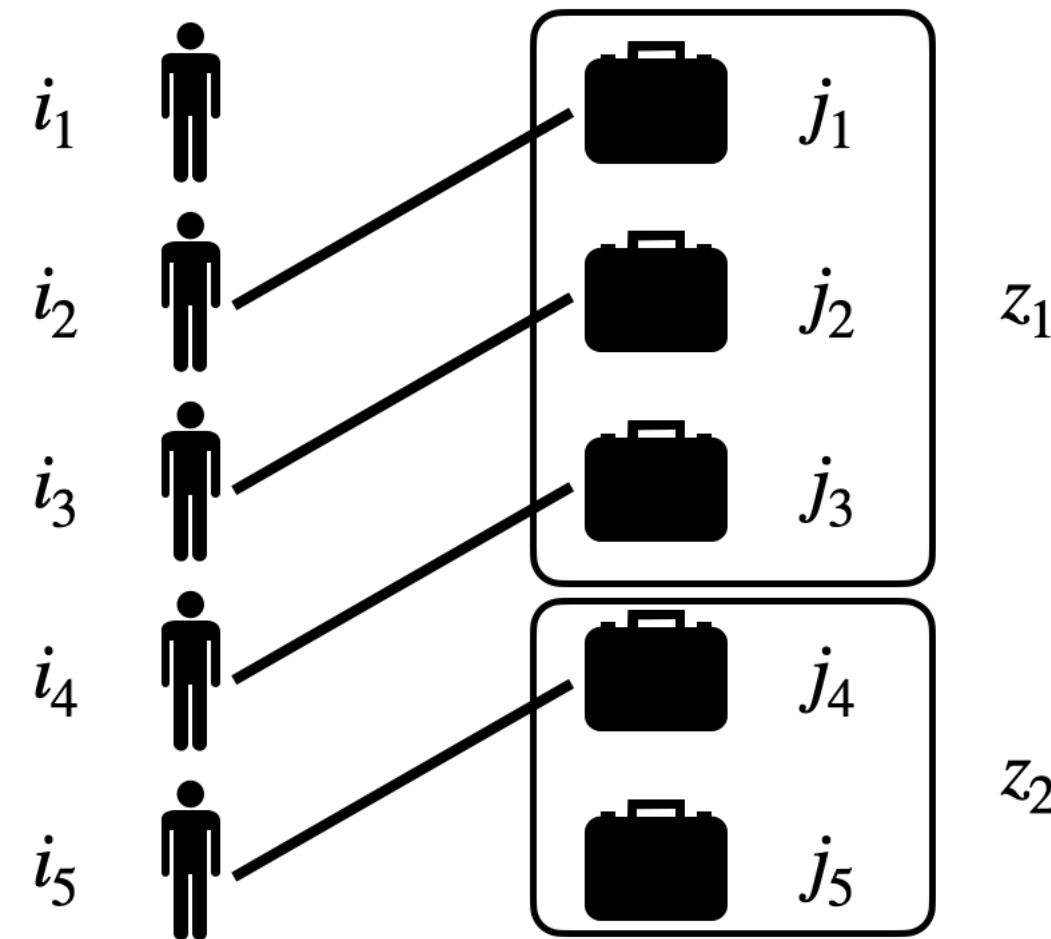
# simulation setup

- we simulate the JRMP market in 2017 using estimated preferences
- compare three different policies:

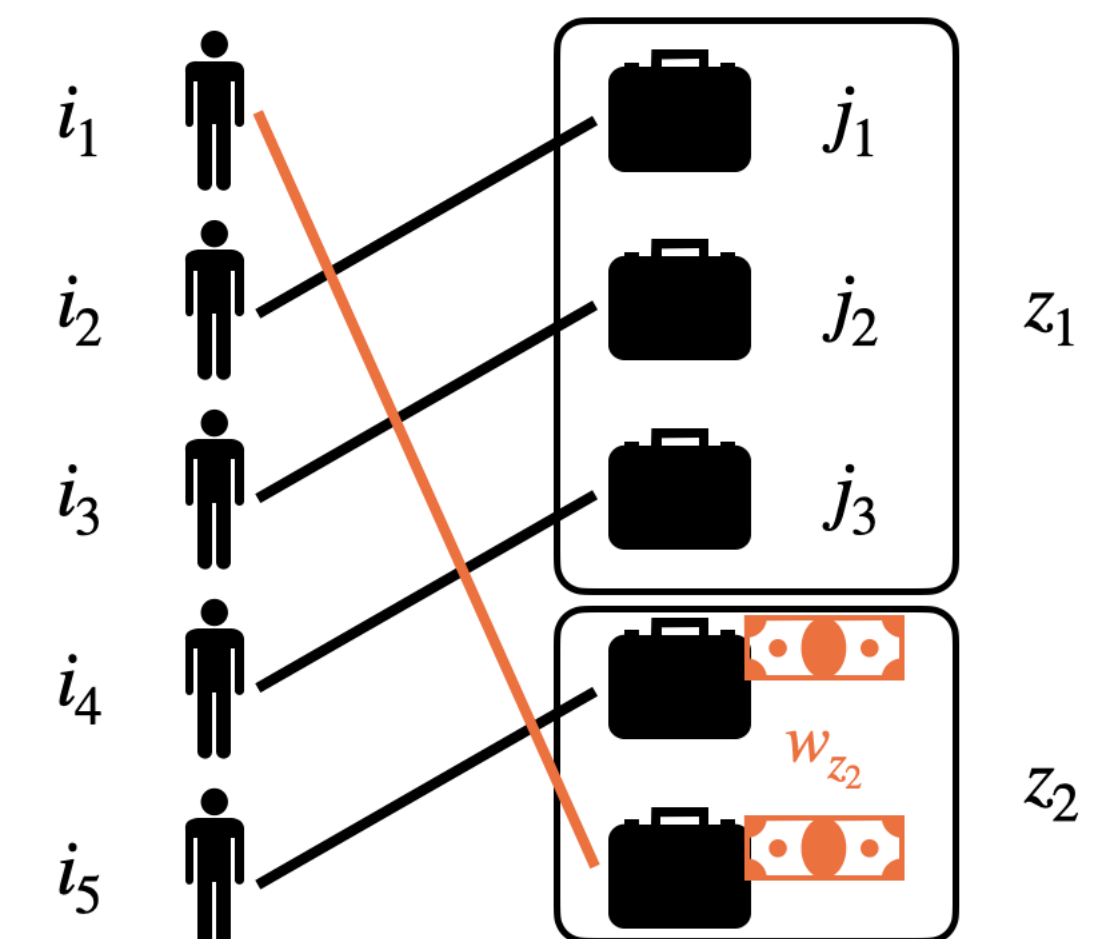
**Artificial Caps (AC)**  
(cap on urban areas)



**No Caps (NC)**  
(all slots available)



**Optimal Subsidy (OS)**  
(all slots available + subsidy)



**regional constraints:** rural prefectures receive  
at least as many residents as they did under the AC



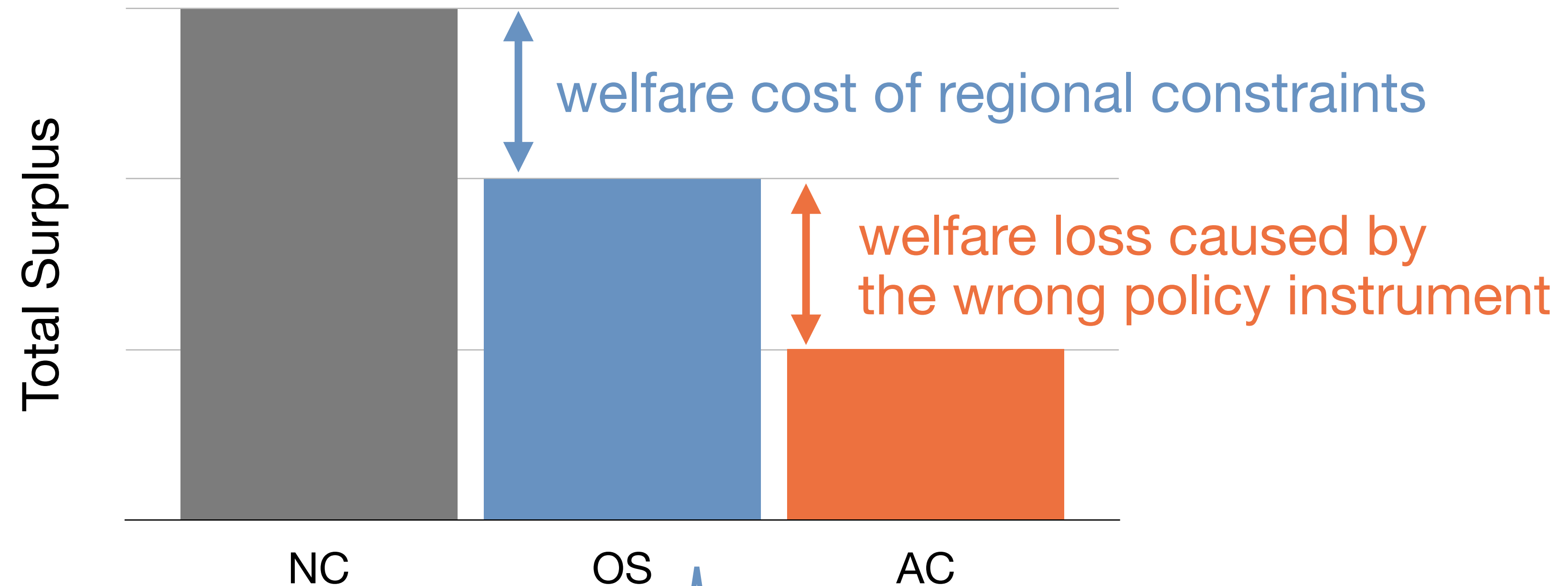
# welfare ranking

**Artificial Caps (AC)**  
(cap on urban areas)

**No Caps (NC)**  
(all slots available)

**Optimal Subsidy (OS)**  
(all slots available + subsidy)

**Fact: NC** maximizes total surplus (w/o regional constraints)



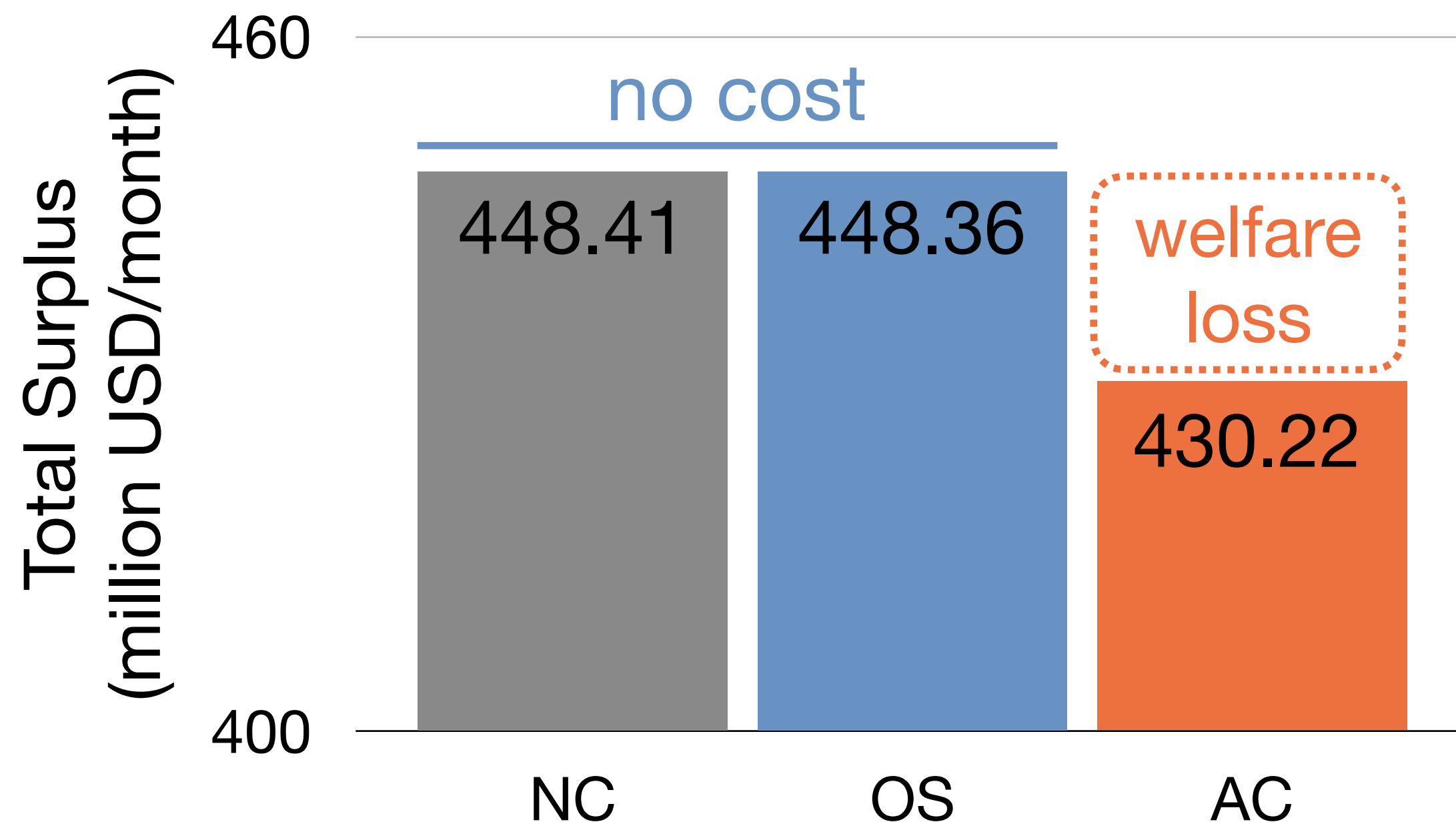
**Thm 1:** **OS** maximizes total surplus s.t. regional constraints

# simulation results

**Artificial Caps (AC)**  
(cap on urban areas)

**No Caps (NC)**  
(all slots available)

**Optimal Subsidy (OS)**  
(all slots available + subsidy)



- the status quo policy (AC) generates a significant welfare loss (\$18M/month)
- the same distributional goal can be achieved by the optimal subsidy policy with almost no cost
- amount of required subsidy is modest
  - \$400/month (10-20% of resident's salary) for each matched pair in rural prefectures
  - total national cost \$100K/month
- caps are blunt instruments that do not account for the intensity of preferences

# conclusion

- we develop a framework to evaluate the efficiency of policies in matching markets
  - optimal taxation policy outperforms any cap-based policy
- we apply the framework to JRMP data:
  - current cap-based policy generates a significant welfare loss
  - modest subsidy can address distributional imbalances, improving social welfare